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ON THE ANALOGY OF CELESTIAL BODIES AND MORE PARTICULARLY
OF ARTIFICIAL EARTH'S SATELLITES
WITH GYROSCOPIC SYSTEMS

by

Robert Genty

(FRANCE)

ON THE ANALOGY OF CELESTIAL BODIES AND MORE PARTICULARLY

OF ARTIFICIAL EARTH'S SATELLITES

WITH GYROSCOPIC SYSTEMS * [Fr. title]*

Transl. into ENGLISH from

ANNOTATIONS: J is a constant $= 0.001\,623\,4$; S is the radius vector of a current point of orbit; S_1 is the semi-great axis of the orbit; S_0 is the terrestrial equatorial radius; $\underline{i}$ is the inclination of the orbit plane to the terrestrial equator; $\underline{n}$ is the mean daily motion of the satellite; μ is the Newtonian Earth's attraction constant; $(v - a)$ is the argument of S; $\underline{a}$ is of ellipse's great axis with the normal to the nod line situated in the orbit plane; ε is the terrestrial constant; φ is the latitude of Υ; $K = \sqrt{\mu S_1 (1 - e^2)}$ is the areas' constant; β is the angle at C of the triangle ABC.

The problems of space raise before astronautical engineers and technicians some delicate problems of orbit modifications and even of orbital plane modifications. However, the various questions concerning the <u>gyroscope</u> are familiar to this personnel, often of aviation background.

It appeared interesting to us to show that the problems of spatial evolution are easier to approach in this way rather than by classical, but far more complex means of Celestial Mechanics.

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* De l'analogie des corps célestes et plus particulièrement des satellites artificiels de la Terre à des systèmes gyroscopiques.]
Note presented by M. Pierre Tardi.

As a matter of example, we shall show hereafter that the calculation of the daily value of the line of orbit nodes of a satellite under the effect of the Earth's equatorial "flange" can be easily conducted in this manner. *

Such calculation method has been used by several authors (Résal, Bruhat, Tardi, etc.) to study the peculiarities of terrestrial rotation under lunar-solar effect, by limiting themselves to principal terms (till one tenth of a second of the arc).

The gyroscopic image remains valid, yet being more delicate for an artificial satellite rotating around the center of the Earth. The kinetic moment of this system is constant and equal to areas' constant $K = \sqrt{\mu S_1 (1 - e^2)}$ and everything goes on as if we had to do with a true gyroscope.

We accept as established the classical formula [1]

$$\Omega = - \mathcal{J} \left(\frac{S_0}{S_1}\right)^2 \frac{n}{(1 - e^2)^2} \cos i$$

and we propose to arrive at that formula by application of the gyroscope theory.

The Figure shows very schematically that the elementary forces of attraction, translating the Newtonian attraction of the "flange" on the artificial satellite, act by their normal components at the orbital plane, downward and at the right of the node line and upward to the left of that line. It results therefrom, that for one convolution of the moving body, there appears a perturbation couple, whose moment is borne by the line of nodes. The extremity of the kinetic moment of the system is thereby animated of a velocity equipollent to this moment, and it thus describes a circle of radius $K \sin i$ in a plane parallel to the equator at the distance $K \cos i$ from the latter. The orbit plane is thus subject to a motion in space, such that its intersection with the equator (line of nodes) rotates around the center of the Earth with an angular velocity

dΩ/dt which thus has to be computed, knowing outright that it has
to be equal to that of the extremity of the kinetic moment. This
angular velocity is easy to obtain by effecting the quotient of the
linear velocity of that point, after all equal to the moment of the
perturbating couple, by the radius of the circle K sin i.

Now it remains to compute the moment of the couple. To that
effect, we make appear at a current point Υ of the trajectory the
prturbation component, normal to orbit plane N, which is obtained
by derivating the second term (latitude function) of the expression
of terrestrial gravitation potential, along the meridian of the point
considered, and by projecting the result of this derivation perpen-
dicularly to the orbit plane. We have for the elementary moment NS:

$$NS = -\frac{\varepsilon}{S^3}\sin 2\varphi\sin\beta = -\frac{\varepsilon}{S^3}\sin 2i\cos v$$

by trigonometrical transformation in the triangle ABC of the Figure.

$\widehat{A} = i$ $\widehat{B} = \frac{\pi}{2}$ $\widehat{C} = \beta$

$AC = V - \frac{3\pi}{2}$ $BC = \varphi$

$AD = \frac{\pi}{2}$ $O\gamma = S$
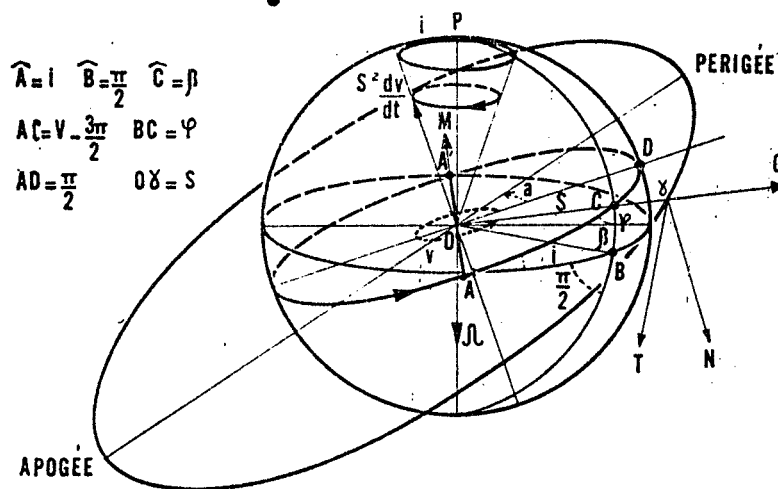
The moment of the perturbing couple being borne by the line
of nodes, it is appropriate to take into consideration only the pro-
jection M of the elementary moment NS on that line, say

$$M = -\frac{\varepsilon}{S^3}\sin 2i\cos^2 v.$$

Transforming this expression with the aid of classical formulae of the Keplerian movement, we have

$$\frac{d\Omega}{dt} = -\frac{2\,\varepsilon\cos i}{\mu S_1^2(1-e^2)^2}\cos^2 v\,[\,1 + e\cos(v-a)]\frac{dv}{dt},$$

expression which we shall integrate while preserving only the term proportional to v and neglecting the periodical terms.

The computation of ε is made as a function of classical formulae with the intervention of the polar and equatorial inertia moments and of the mass of the Earth, together with the universal gravitational constant. Once all calculation operations are completed, we have, by giving v the value corresponding to 24 hours, say in degrees : n , the mean diurnal movement of the satellite (so as to permit arriving at formual (1)):

$$\Omega_{\text{degrees/day}} = -\,0.00.\,634\,2\left(\frac{S_0}{S_1}\right)^2\frac{n}{(1-e^2)^2}\cos i.$$

· The gyroscopic analogy becomes an identity in the case of a circular orbit, which is a frequent case in the current spatial field.

*** THE END ***

Translated by ANDRE L. BRICHANT

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REFERENCE

[1]. - BROWER AND KOZAI, Astronom. J., No.1274, November 1959.

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